Categorical quantum models and logics

Chris Heunen

7 januari 2010

Welcome, thank you for coming.
During the next hour I will defend my dissertation.

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Such an official ceremony is no place for experimental presentations, but nevertheless ...

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I will subtitle myself while introducing what the dissertation is about.

Categorical quantum models and logics

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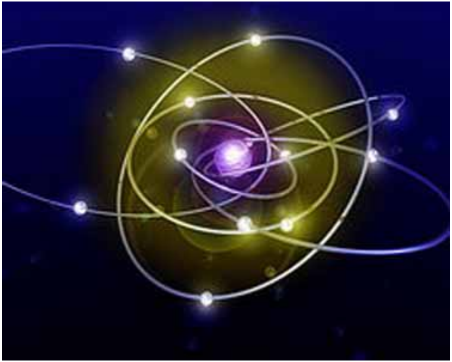
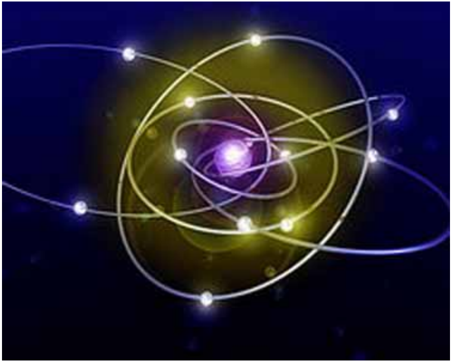
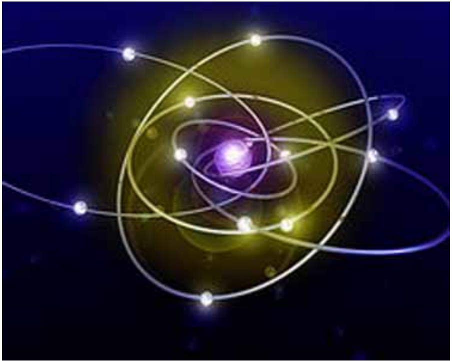
Let's start with 'quantum', perhaps the most intimidating word in the title.



Quantum mechanics is the best description of nature on small scales that we have today.



It is a very odd description: if one zooms in very far, nature's behaviour is beyond our intuition.



Why? Our intuition for what is odd, and what isn't, is acquired on the much larger scale of everyday life.

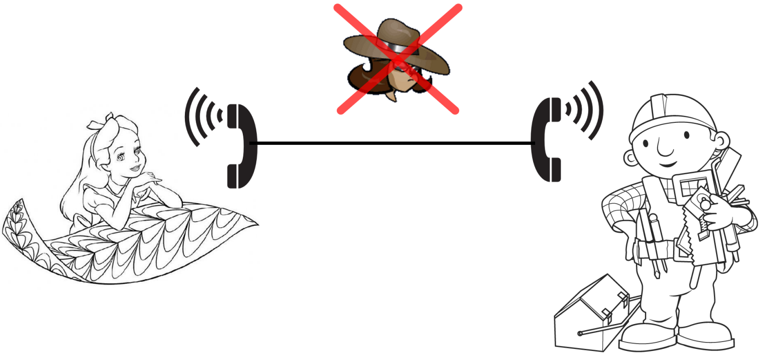
For example, we find it normal that one cannot walk on water, or carry it in one's hands.

But if one is small enough, like these merry flies,
that is not so odd anymore at all.

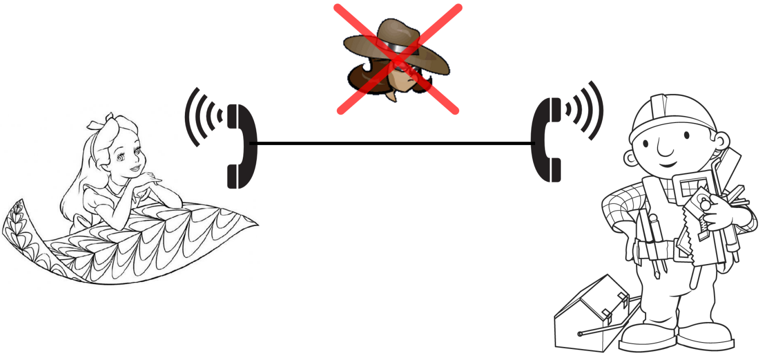
On the still smaller scale on which quantum mechanics reigns, there are still more fundamental oddities,

but this example indicates that odd things can occur when one becomes smaller.

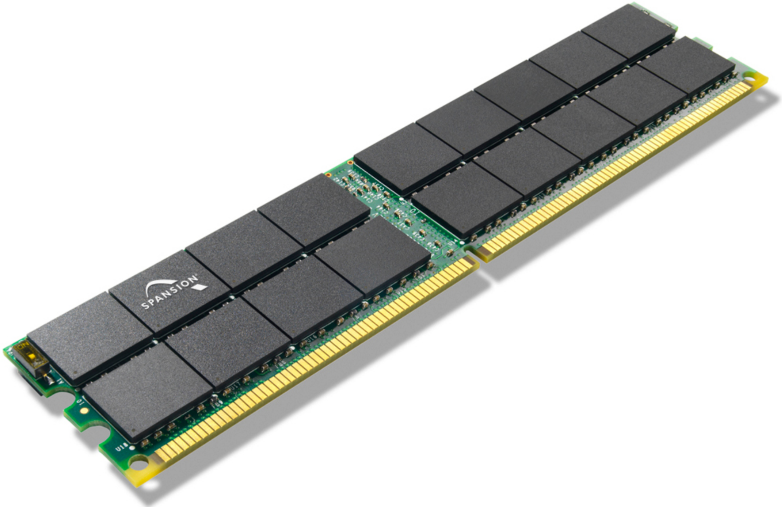
This fundamental oddity of small scales has advantages.
By using it a phone line can be made which can detect eavesdropping.



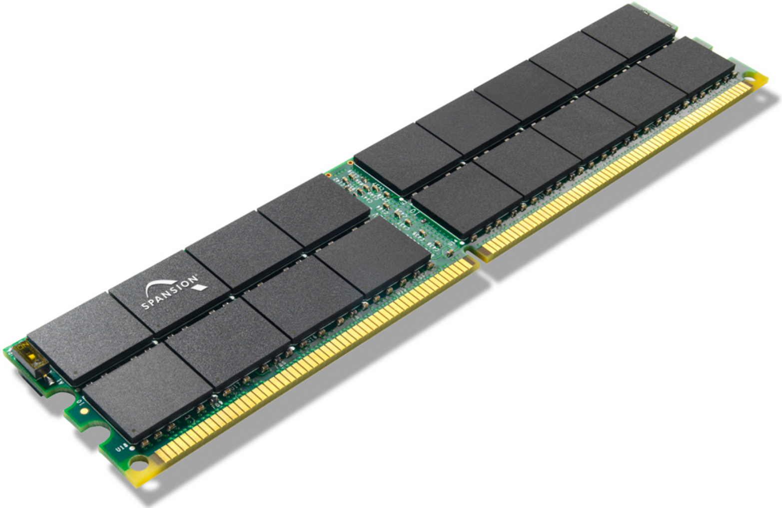
Also, computers can be made that solve certain problems essentially faster than current computers.



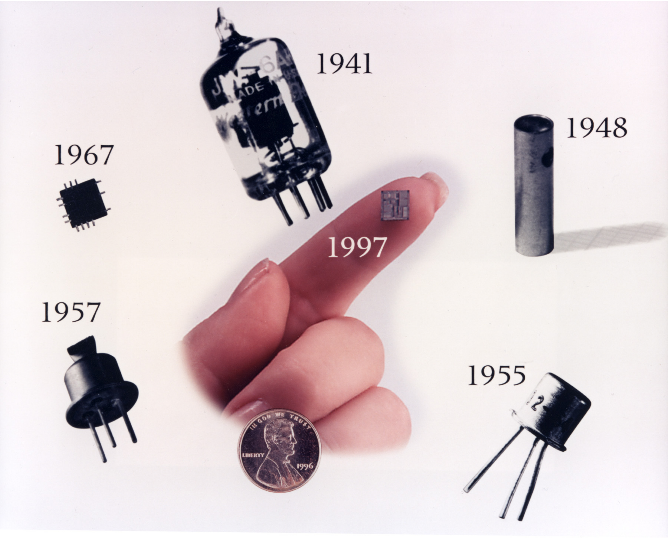
Such socalled quantum computers use principles that fundamentally differ from those of current computers.



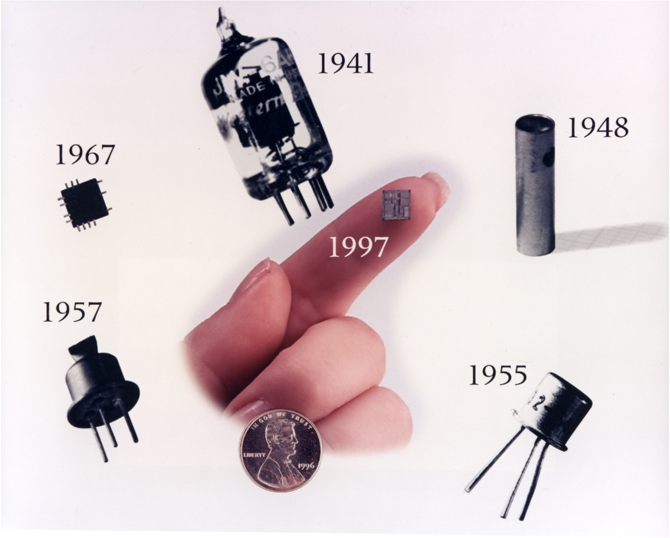
Computer memory consists of bits; units that are either 0 or 1. Today, a single bit takes approximately 30 nanometers.



Those dimensions change. Previous bits were bigger.
Future bits will be smaller.



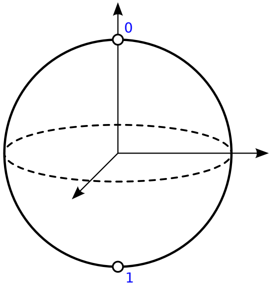
Some day they will be so small that quantum mechanics comes into play.



If one shrinks a bit enough, it becomes a qubit.
When measured, its value is still either 0 or 1.

bit

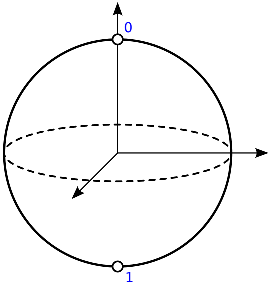
qubit



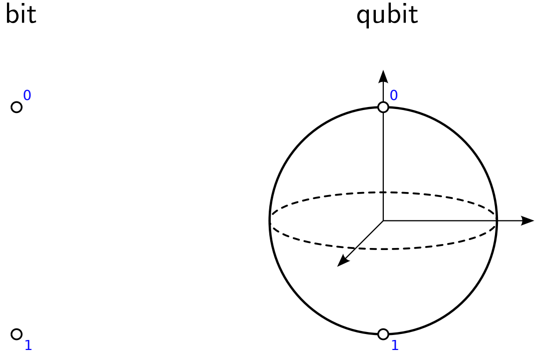
But left unmeasured, its value can be something else entirely.
In general, its state space is a sphere instead of two isolated points.

bit

qubit

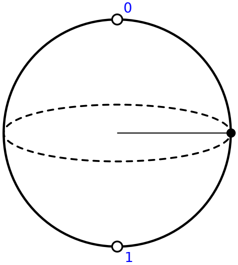


This entails that strange things can happen.



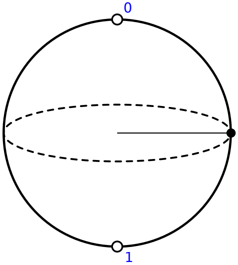
For example, a qubit in this state will give an outcome upon measurement of 0 half of the time, and 1 the other half.

qubit in superpositie



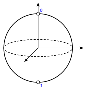
In a certain sense the qubit is 0 and 1 at the same time, though that formulation is misleading.

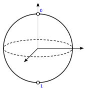
qubit in superpositie

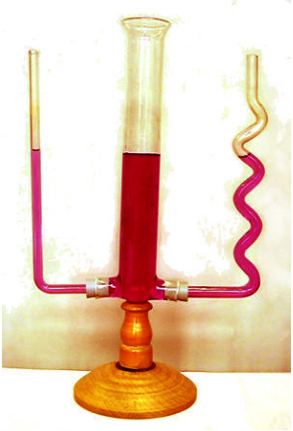


A similar oddity happens when considering not one but two qubits. These can be entangled in such a state, that ...

entanglement

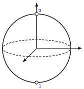


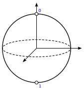


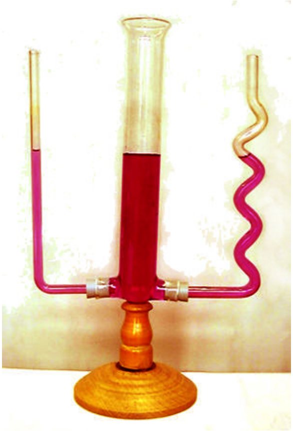


if one is measured and gives outcome 0, then the other must give 1. This holds instantaneously, even if the two qubits are miles apart.

entanglement

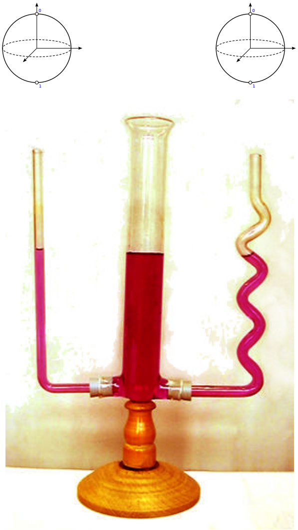






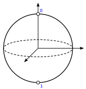
The water in the two ends of a bent pipe also has this property, but that is so because they form communicating vessels.

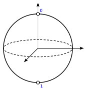
entanglement

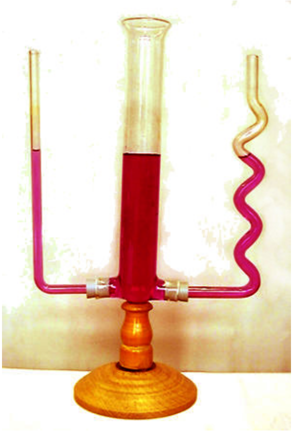


The odd thing about entangled qubits is that this behaviour occurs without a common cause.

entanglement







The third oddity I want to mention is (non)commutativity, which means it is not sensible to measure certain properties of qubits simultaneously.

(niet-)commutativiteit

$\not\leftrightarrow$

This is caused by the fact that the order of measurements matters.
The effect of undressing and then taking a shower ...

(niet-)commutativiteit

$\not\leftrightarrow$

is completely different than vice versa!
Thus there are also disadvantages to using quantum mechanics:

(niet-)commutativiteit

$\not\leftrightarrow$

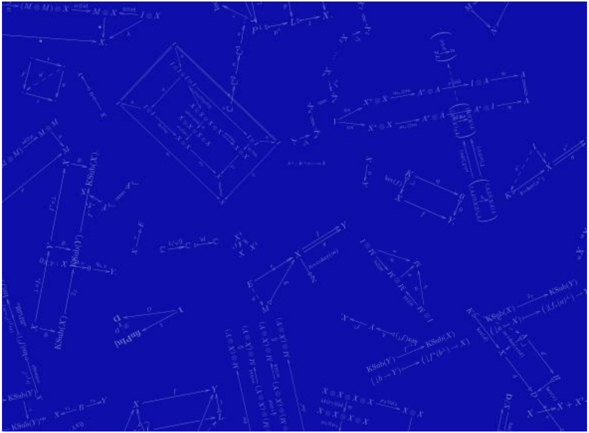
one's intuition will block attempts at understanding.
It is better to reason purely mathematically.

(niet-)commutativiteit

$\not\leftrightarrow$

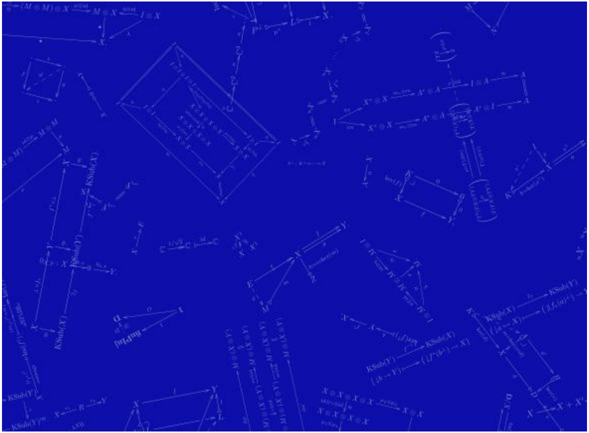
Traditionally one does so by making assumptions about the set of states. For a qubit, this is a sphere. The first half of the dissertation follows ...

categorie



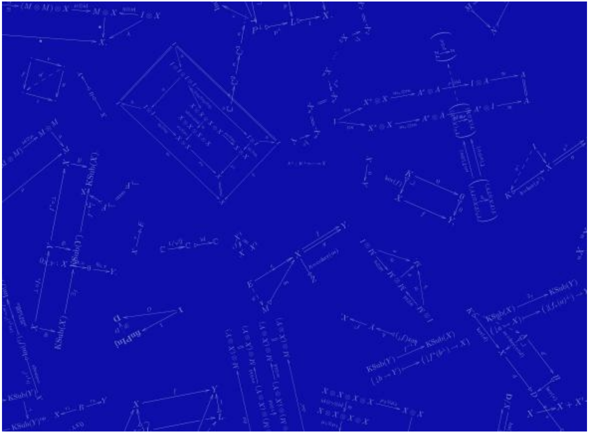
a different approach: let's not be modest, and study
all possible state spaces at once, and all relationships between them.

categorie



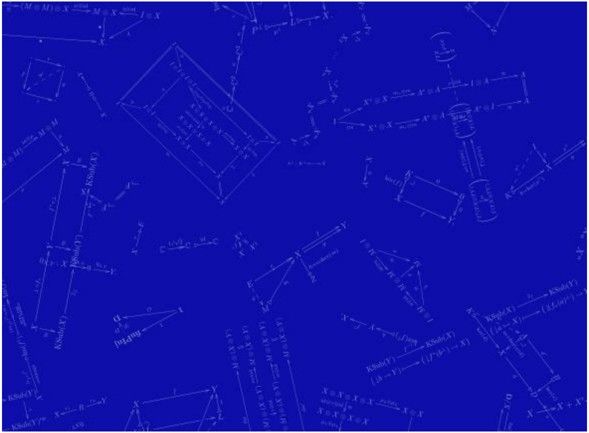
This is called a category, and is depicted on the cover and virtually every page of the thesis:

categorie



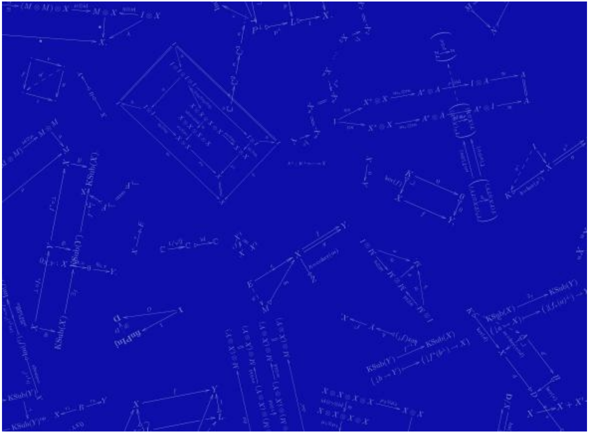
such diagrams concern connections between objects,
not so much the objects themselves.

categorie



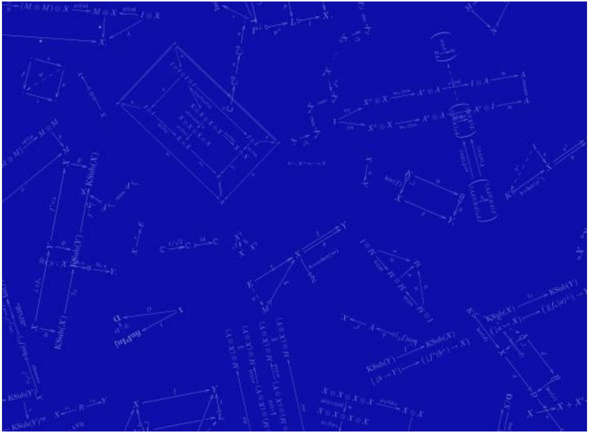
Instead of assuming some internal structure of a quantum system, we study how the system relates to others.

categorie



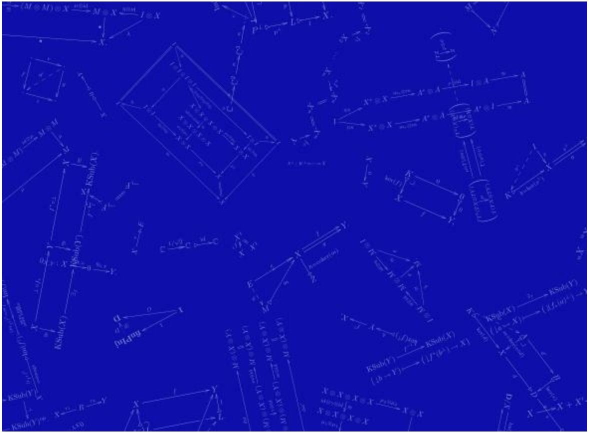
After all, we don't know what the internal structure is!
One might say we elect a sociological approach over a neurological one.

categorie



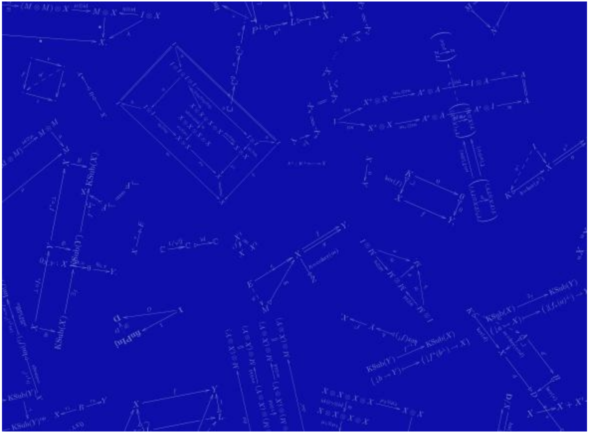
Instead of trying to find out how one individual's brain works, we only consider how that person behaves in society.

categorie



Thus we can qualitatively see what properties cause quantum behaviour.

categorie



For example, an important such property is that every relation between quantum systems is invertible: if system A is somehow related to B, ...

axioma's

$$A \xrightarrow{f} B$$

$$A \xleftarrow{f^\dagger} B$$

then B is also connected to A. Thus one can make an assumption about the category for every behaviour obtained from physical experiments.

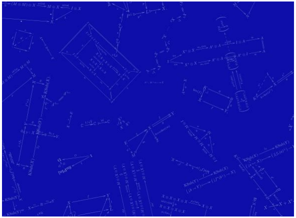
axioma's

$$A \xrightarrow{f} B$$

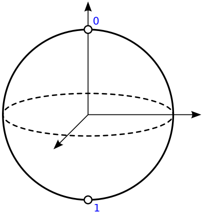
$$A \xleftarrow{f^\dagger} B$$

Chapter 3 shows that if one assumes the three oddities we saw in this way, then the category always embeds into the traditional physical model.

inbedding

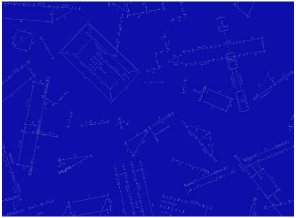


$\longrightarrow$

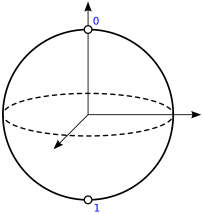


In a sense this justifies the traditional model of quantum mechanics.

inbedding

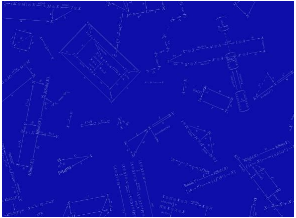


$\longrightarrow$

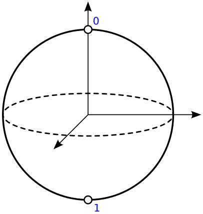


It turns out that (complex) numbers emerge in every model of quantum mechanics, even if not explicitly assumed.

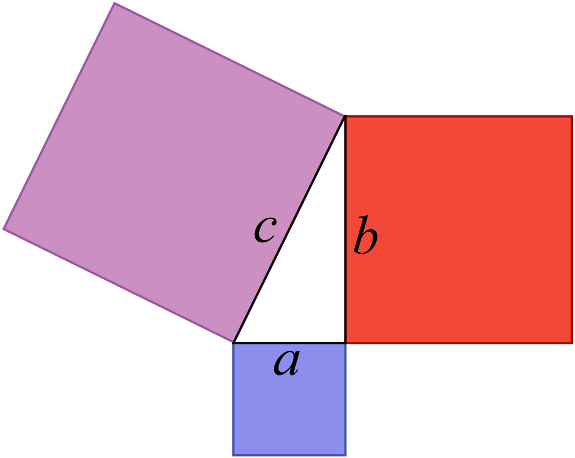
inbedding



$\longrightarrow$

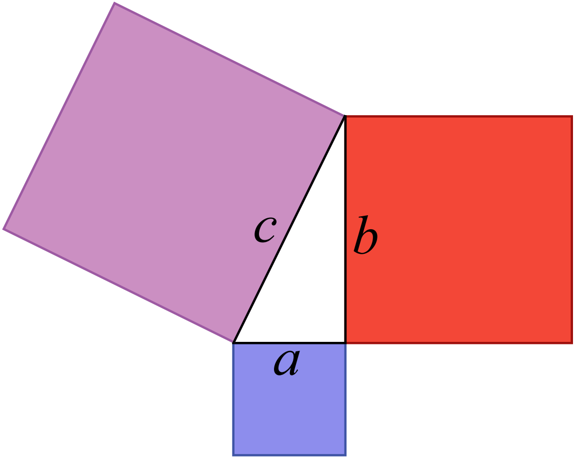


The second half of the thesis nonetheless tries to make sense of the oddities. That is, we try to set up a logic for quantum mechanics.



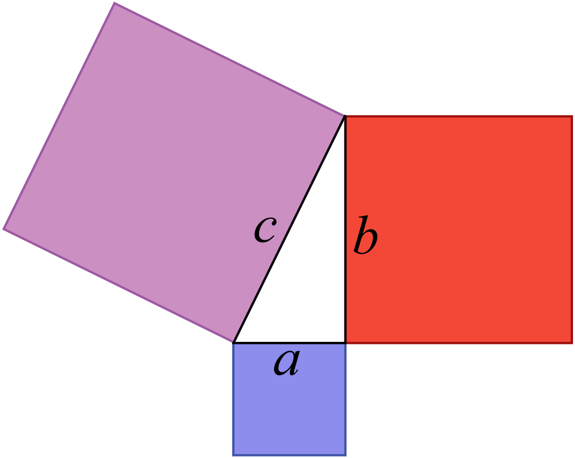
$$a^2 + b^2 = c^2$$

The formal mathematics that we replace naive intuition with, relies on the notion of proof.



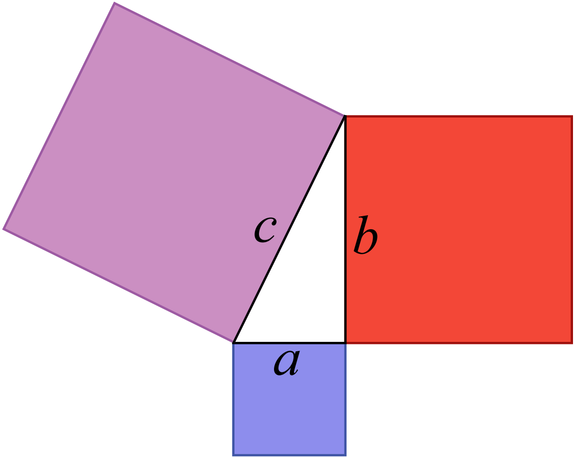
$$a^2 + b^2 = c^2$$

I mean a different kind of proof than you might know, from, say, the law. I mean, for example, proof of the well-known Pythagorean theorem.



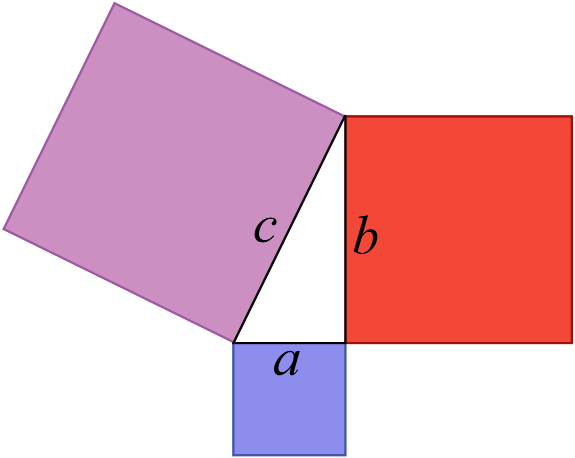
$$a^2 + b^2 = c^2$$

This is not something that has been tested on 100 right-angled triangles, and happened to hold for most.



$$a^2 + b^2 = c^2$$

No; from purely logical deductions, it is certain that the theorem holds for any right-angled triangle you will ever encounter.



$$a^2 + b^2 = c^2$$

Here is a very simple example of such a proof.
It is raining. When it rains, one gets wet. Hence I get wet.

Het regent.
Als het regent, word je nat.
Dus ik word nat.

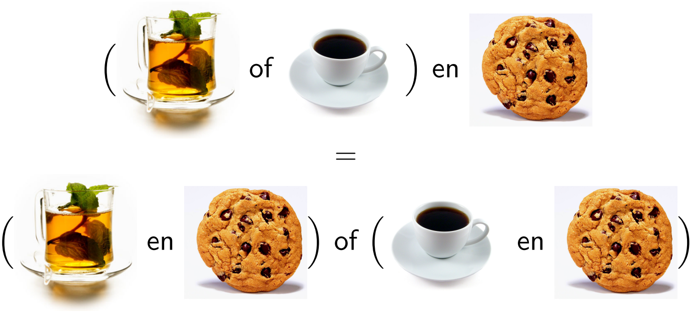
In symbols: A and $A \Rightarrow B$, hence B.
Notice that logic is the grammar of a kind of language.

A en $(A \Rightarrow B)$, dus B.

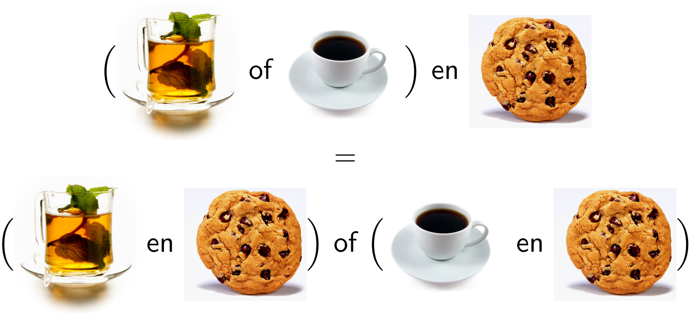
Another such grammar rule could be:
(A or B) and C = (A and C) or (B and C).

$$(A \text{ of } B) \text{ en } C$$
$$=$$
$$(A \text{ en } C) \text{ of } (B \text{ en } C)$$

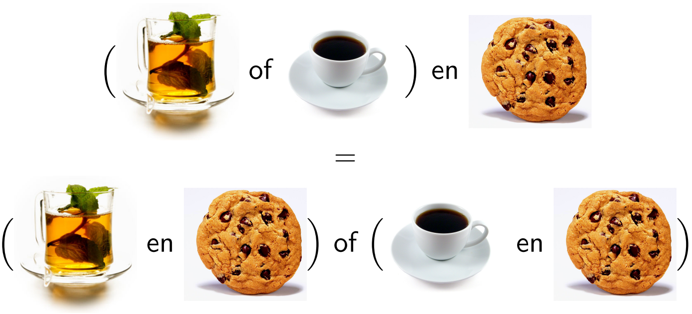
If offered tea or coffee with a biscuit,
one expects either tea with a biscuit, or coffee with a biscuit.



But because of the oddities of quantum mechanics, this no longer holds! There are properties of qubits for which this equation doesn't hold.



Nevertheless this is traditionally called 'quantum logic'. Chapter 4 shows that this socalled logic holds in our categorical models unabated.



But one cannot in good faith call something this unintuitive logic.
That is why I try something else in chapter 5.

We play a technical trick there: by altering the grammar rules of one's logic, one can pretend that a quantum system is intuitive.

Imagine for the moment that our world is Smurf village, and hence that you are a Smurf.

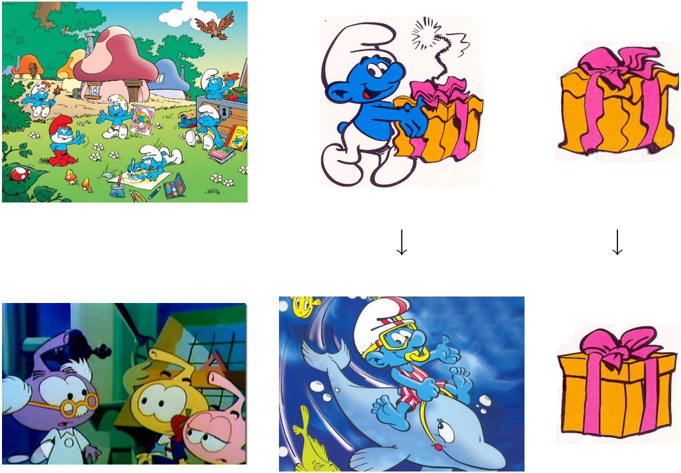
The world contains quantum systems, that we cannot intuitively understand.
In the cartoon, a quantum system is a package, that looks mysterious.

From the right perspective, that package would appear normal.
If we were Snorks instead of Smurfs, and hence lived under water ...

we would have seen through the water ripples,
and the package had appeared perfectly normal.

$\downarrow$

Now, Smurfs are no Snorks. But a Smurf can live in Snorkland just fine. All he has to do is wear goggles and a snorkel, and forget he is a Smurf.



Thus odd things can be made normal by “changing world”.
That is the main trick of chapter 5.

↓ ↓

Given a single quantum system, we make a world we can live in just fine which has fine logical laws, and in which the system looks perfectly normal.

↓ ↓

If we alter the grammar rules of our logic thus, oddities can be made normal. To 'understand' quantum mechanics, one must speak the right language.

$\downarrow$ $\downarrow$

Clearly this is a crude simplification, like other analogies I used before. At some points what I said is not even entirely correct.

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But hopefully the topic of my dissertation is now somewhat clear.
At least you can now interpret words of the title!

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